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Simple correlations for saturated liquid and vapor densities of pure fluids

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Abstract

Two correlations for the saturated liquid and vapor densities of pure substances particularly those used as working fluids in refrigeration machines are proposed. They are shown to represent well the reduced density as a function of the reduced temperature ranging from 0.5 to 1. For about 30 pure substances with acentric factor and critical compressibility factor varying in a wide range, the correlations predict accurately the saturation densities.

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1. Introduction

The knowledge of the physical properties of fluids is essential to the equipment design of the refrigeration machines. In particular, the quantitative description of vapor–liquid equilibrium (VLE) is required in such applications.

Equations of state (EOS) are widely used for VLE calculations of pure and mixtures of non-polar and slightly polar substances. Such equations inherently provide good information but, at present, their usefulness to polar substances is limited [1,2]. Many successful equations for densities at saturation of pure fluids had been developed following the general form [3,4]:

$$
\rho_{\rm r} = \sum_{i=1}^{p} a_i T^i \tag{1}
$$

$$
\vec{a} = \{a_1, a_2, \ldots, a_p\}
$$

where $\vec{a} = \{a_i\}$ is a vector of adjustable parameters and *i* are real powers.

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Although the general correlation (1) predicts with good accuracy the saturated densities data, it is a multiparameter model with power's values depending on the chosen compound that cannot be generalized to all considered fluids to obtain a simple and a common correlation.

The proposed research aims a principal objective: the elaboration with the help of the corresponding state principle of simple correlations for the saturated vapor and liquid densities of a set of pure fluids, especially of those used—or proposed for use—as working fluids in absorption refrigeration machines like water, ammonia, carbon dioxide, hydrocarbons, etc. Because the models will be integrated in a simulation software for refrigeration machines, the correlations do not need to describe with equal good accuracy the entire temperature range of saturated liquid and vapor densities, but would provide acceptable results in the working range of the previous machines.

About 30 fluids with an acentric factor, ω , and a critical compressibility factor, *Z*_c, ranging, respectively, from −0.2 to 0.35 and 0.23 to 0.3 are investigated. Their main characteristics are given in Table 1. The proposed models for the reduced saturated vapor and liquid densities are expressed in reduced variables: $T_r = T/T_c$ and $\rho_r = \rho/\rho_c$, where T_c and ρ_c are the critical data.

Adjusta[ble param](#page-1-0)eters of the new models are obtained with the help of a nonlinear regression procedure [5] by minimizing as objective function the sum of the quadratic

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average deviation between the density data [6] and its calculated values:

$$
\Phi = \sum_{i=1}^{N} \left(1 - \frac{\rho_{\text{r, calculated}}}{\rho_{\text{r}}} \right)_{i}^{2} \tag{2}
$$

Table 1

Characteristics of the considered fluids [6]: critical data, acentric factor, nor[mal](#page-8-0) [bo](#page-8-0)iling point temperature, temperature ranges of validity, *T*min, *T*max and number of used data

Fluid	T_c (K)	P_c (bar)	ρ_c (mol/l)	ω	$T_{\rm nbp}$ (K)	$T_{\min} - T_{\max}$ (K)	$\cal N$
H ₂	33.19	13.15	14.94	-0.2140	20.39	$16.00 - T_c$	18
D_2	38.34	16.65	17.33	-0.1750	23.31	19.00 $-T_c$	20
Ne	44.49	26.79	23.88	-0.0387	27.10	$25.00 - T_c$	$21\,$
Ar	150.69	48.65	13.41	-0.00219	87.30	$84.00 - T_c$	68
Kr	209.48	55.10	10.84	-0.0017	119.78	$116.00 - T_c$	95
Xe	289.73	58.40	8.37	0.0036	165.03	$162.00 - T_c$	28
CH ₄	190.56	4.60	10.14	0.0114	111.68	$92.00 - T_c$	50
O ₂	154.58	50.43	13.63	0.0222	90.19	$75.00 - T_c$	81
$\rm N_2$	126.25	33.98	11.18	0.0372	77.36	63.15 $-T_c$	23
F ₂	144.41	51.72	15.60	0.0449	85.04	$72.00 - T_c$	38
CO	132.80	34.93	10.85	0.0510	81.63	69.00 $-T_c$	33
C_2H_4	282.35	50.42	7.64	0.0866	169.38	$142.00 - T_c$	72
C_2H_6	305.33	48.72	6.87	0.0993	184.55	$152.00 - T_c$	77
NF ₃	234.00	44.61	7.92	0.1260	144.14	$117.00 - T_c$	31
C_3H_6	365.57	46.65	5.31	0.1408	225.46	$184.00 - T_c$	47
C_3H_8	369.85	42.48	4.96	0.1524	231.06	$186.00 - T_c$	47
$i - C_4H_{10}$	407.85	36.40	3.86	0.1850	261.48	$200.00 - T_c$	43
$n - C_4H_{10}$	425.16	37.96	3.92	0.2000	272.60	$210.00 - T_c$	44
R ₂₂	369.29	49.90	6.06	0.2208	232.34	$185.00 - T_c$	37
CO ₂	304.21	73.84	10.62	0.2239	194.75	$220.00 - T_c$	23
C_5H_{12}	469.70	33.66	3.22	0.2510	309.21	$235.00 - T_c$	48
NH ₃	405.40	113.33	13.21	0.2560	239.82	$200.00 - T_c$	42
R143a	345.86	37.61	5.13	0.2615	225.91	$172.00 - T_c$	36
R152a	386.41	45.17	5.57	0.2752	249.127	$191.00 - T_c$	40
R32	351.26	57.82	8.15	0.2769	221.50	$172.00 - T_c$	37
R123	456.83	36.62	3.60	0.2819	300.98	$228.00 - T_c$	47
R124	395.42	36.24	4.10	0.2881	261.19	$195.00 - T_c$	42
C_6H_{14}	504.82	30.18	2.71	0.2990	341.86	$250.00 - T_c$	51
R125	339.33	36.29	4.78	0.3052	225.06	$173.00 - T_c$	35
R134a	374.24	40.59	5.02	0.3268	247.08	$185.00 - T_c$	39
H_2O	647.10	220.64	17.87	0.3443	373.12	$325.00 - T_c$	66
C_7H_{16}	540.13	27.27	2.32	0.3490	371.53	$270.00 - T_c$	54

The predictive quality of the proposed correlations is evaluated by means of the root standard deviations RMSD and RMSDr:

$$
RMSD = \left(\sum_{i}^{N} \frac{(\rho_{r} - \rho_{r, calculated})_{i}^{2}}{N}\right)^{1/2}
$$
 (3)

RMSDr% =
$$
100 \left(\frac{\sum_{i}^{N} (1 - \rho_{r, calculated}/\rho_r)_i^2}{N} \right)^{1/2}
$$
 (4)

with *N* being the number of used data.

2. Correlations

Pressure–volume–temperature (*PVT*) equation of state of pure gases and liquids are usually used to describe the volumetric behavior of the pure substances. Two-parameter cubic equations of state (CEOS) had been very successful for non-polar and slightly polar components. Their common shortcomings are inaccuracy of the predicted liquid density [7] and a unique value of the critical compressibility factor Z_c [2]. The virial equation can represent only

Fig. 1. Reduced saturation liquid density in the $(\beta(\tau), \ln(\rho_{r,f}))$ plane. Fig. 2. $\omega(a + b)$ variation vs. acentric factor ω .

modest deviations from ideal gas behavior. Although the Lee–Kesler (LK) and the more elaborated EOSs (BACK, PC-SAFT, etc. [9]) are applicable over broader ranges of temperature and pressure and are capable of representing liquid-phase behavior, many specific computational problems are encountered when they are used [8].

2.1. Saturated liquid density

The standard equation to esti[mate](#page-8-0) saturated liquid density is the correlation of Rackett [1]:

$$
\rho_{\rm f} = \left(\frac{RT_{\rm c}}{P_{\rm c}}\right) Z_{\rm c}^{-(1+\tau^{2/7})} \tag{5}
$$

where τ is given [as](#page-8-0)

$$
\tau = 1 - T_{\rm r} \tag{6}
$$

To enhance the predictive capabilities of (5), Spencer and Danner proposed to replace the critical compressibility factor of the fluid, Z_c , by an adjustable parameter, Z_{RA} , the Rackett compressibility factor [1,8]

$$
\rho_{\rm f} = \left(\frac{RT_{\rm c}}{P_{\rm c}}\right) Z_{\rm RA}^{-(1+\tau^{2/7})} \tag{7}
$$

where Z_{RA} is the [only](#page-8-0) [su](#page-8-0)bstance-specific constant in the equation. Sample values of Z_{RA} found by using (2) are listed in Table 2. Correlation (7) predicts accurately the saturated liquid density with an overall RMSDr of 0.55%, except [for](#page-5-0) polar substances like water where it is of about 2.25% (cf. Table 2).

The models we proposed are intended to reproduce equally well the saturation data of all considered fluids polar as well as non-polar. The saturated liquid density is expressed as a function of the reduced temperature, *T*r, for each fluid. Fig. 1 shows that $[\ln(\rho_{r,f})]$ can be expressed with the help of the intermediate function $\beta(\tau)$:

$$
\ln(\rho_{r,f}) = a(\beta - \exp(1 - \beta))\tag{8}
$$

[with](#page-3-0)

$$
\beta(\tau) = 1 + \tau^b \tag{9}
$$

Besides its simplicity—only two adjustable fluid-specific parameters (*a* and *b*) are involved—the model fulfills the requirement of a steep decrease of the liquid density in the close vicinity of the critical point [3]:

$$
\lim_{T \to T_c} \left(\frac{\partial \rho_f}{\partial T} \right) = -\infty \tag{10}
$$

This constraint is equi[valen](#page-8-0)t to the two conditions $ab > 0$ and $b < 1$.

2.1.1. Results and discussion

Correlation (8) can be considered either as a two or as a one adjustable parameter equation. If the constants *a* and *b* are treated separately, it is a two-parameter equation

with an overall RMSDr of 0.23% (cf. Table 2). If, on the other hand, they are both correlated to the acentric factor ω (Fig. 2)—then they are dependent on one another—we found that

$$
\frac{1}{\omega(a+b)} = a_1 + a_2\omega^3 + \frac{a_3}{\omega} \tag{11}
$$

[w](#page-3-0)here a_1 , a_2 and a_3 are constants given in Table 3. In this case, the overall RMSDr increases to 0.45%.

Parameter *b* in Eq. (9) could not be correlated well with the acentric factor ω (Fig. 3) or the critical compressibility factor *Z*c; if it is done, the inaccuracy [of the pr](#page-5-0)edicted saturated density increases strongly.

2.2. Saturated vapor density

The same gases of Table 1 are considered here, except hydrogen and deuterium.

An equation for $\rho_{r,v}$ as a function of the temperature is established by plotting, for each fluid, $[\ln(\rho_{r,v})]$ versus $\alpha(\tau)$, a function o[f the redu](#page-1-0)ced temperature

$$
\alpha(\tau) = \exp(\tau^{1/3} + \tau^{1/2} + \tau + \tau^m)
$$
 (12)

where $[\ln(\rho_{r,v})]$ is found to depend on $\alpha(\tau)$ as follows (Fig. 4):

Fig. 3. *b* variation vs. acentric factor ω .

Fig. 4. Reduced saturation vapor density in the $(\alpha(\tau), \ln(\rho_{r,v}))$ plane.

$$
\ln(\rho_{r,v}) = p[\alpha(\tau)^n - \exp(1 - \alpha(\tau))]
$$
\n(13)

2.2.1. Results and discussion

Like Eq. (8), model (13) is made to fulfill the requirement of a steep increase of the density in the vicinity of the critical region:

$$
\lim_{T \to T_c} \left(\frac{\partial \rho_v}{\partial T} \right) = +\infty \tag{14}
$$

which implies if $m > 0$, then $(-p)(n + 1)$ must also to be.

It is found that the parameters *n* and *p* are correlated to the acentric factor ω through the simple relation (Fig. 5):

$$
\frac{1}{\omega(n-p)} = \frac{n_1 + n_2}{\omega} \tag{15}
$$

Table 3 Constants a_1 , a_2 and a_3 in Eq. (11)

The values of the parameters n_1 and n_2 are given in Table 4. Correlation (13) can be viewed as a two adjustable parameter equation (*m* and *n* or *m* and *p*) but can also be reduced to just a one parameter equation when the parameter *p* is expressed as function of the critical compressibility factor *Z*^c

$$
\frac{Z_c}{p} = \frac{p_1 + p_2 Z_c \ln(Z_c) p_3}{Z_c} \tag{16}
$$

where p_1 , p_2 and p_3 are three adjustable constants as given in Table 4.

Thus, model (13) can be viewed either as a three-, a twoor just a one-parameter equation, depending on whether correlations (15) and (16) are considered or not. In either case, the proposed correlation reproduces accurately the vapor density at saturation. For instance, in the first case (three

Fig. 6. Critical compressibility factor Z_c vs. acentric factor ω .

Fig. 7. p variation vs. acentric factor ω .

Fig. 8. m variation vs. acentric factor ω .

parameters) the overall RMSDr is under 1% (0.98%) (cf. Table 5). Its value is 1.25% (cf. Table 5) in the one-parameter case.

In view of the obtained results some general remarks can be formulated:

- Because of the no[n-regular](#page-6-0) behavior of Z_c on ω (Fig. 6) for the considered fluids, parameter *p* could not be expressed as a simple function of the acentric factor ω (Fig. 7).
- Although a correlation between *m* and the acentric factor ω (Fig. 8) gives a three-parameter [corresp](#page-7-0)onding state equation, the accuracy of the model decreases this way.
- For each fluid, the inaccuracy of th[e predic](#page-7-0)ted vapor density at saturation increases substantially as the critical re[g](#page-7-0)ion approaches.

3. Conclusion

The vapor–liquid envelope of water and ammonia are shown in Figs. 9 and 10, respectively, in comparison with the saturated data. The good agreement observed illustrates the quality of the proposed analytical expressions for the liquid and vapor density. Thus the proposed models—developed

Fig. 9. $\rho_{r,f}$, $\rho_{r,v}$ vs. τ for H₂O.

Fig. 10. $\rho_{r,f}$ and $\rho_{r,v}$ vs. τ for NH₃.

for the use in the refrigeration machines's simulation software—are accurate, simple and can be used for polar as well as non-polar fluids.

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